

Periodic distribution of galaxies in generalized scalar tensor theory

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With the help of Nordtvedt's scalar tensor theory an exact analytic model of a non-minimally coupled scalar field cosmology in which the gravitational coupling G and the Hubble factor H oscillate during the radiation era is presented. A key feature is that the oscillations are confined to the early stages of the radiation dominated era with G approaching its present constant value while H becoming a monotonically decreasing function of time. The Brans Dicke parameter ω is chosen to be a function of Brans Dicke scalar field so that no conflict with observational constraints regarding its present value arises.

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I. INTRODUCTION

One of the most striking observations in recent times having a cosmological implication has been that of a periodicity in the distribution of galaxies. This possibility was pointed out by Tifft [1] in 1977 and was subsequently confirmed by the deep pencil beam survey directed at the galactic pole as reported by Broadhurst, Ellis, Koo and Szalay (BEKS) in 1990 [2]. The successive peaks of the galaxy distribution has been found to have a periodic separation of about $128h^{-1}$ Mpc observed over a scale of $2000h^{-1}$ Mpc (here h is the present value of the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$).

A spatial periodicity of this nature is indeed a bewildering and uncomfortable feature because in the standard cosmological models a spatial homogeneity (the cosmological principle) is built in. Naturally there have been attempts to account for this discovery in terms of an illusion introduced by a temporal periodicity in cosmological quantities like the Hubble's parameter H or the Newton's constant G , rather than a real spatial inhomogeneity. The first serious attempts towards this came immediately following the BEKS results through the work of Morikawa [3,4] and that of Hill, Steinhardt and Turner [5]. Morikawa introduced a scalar field, having a tiny mass scale of order 10^{-31} eV [4], non-minimally coupled to gravity and showed that the required periodicity could possibly be obtained from this model. This ansatz has its problems. For example, the predicted value of q_0 , the present deceleration parameter, from this ansatz, is positive and could be as high as 60. This squarely contradicts the recent observation of a presently accelerating universe [6]. Hill *et al.* [5] discussed the possibilities of explaining the BEKS results via different scenarios. An oscillating dark matter field or an oscillating Rydberg "constant" were shown to be inconsistent but an oscillating G or an oscillating galactic luminosities could potentially solve the problem. Busarello *et al.* [7] showed that a non-minimally coupled scalar field can indeed produce an oscillation consistent with the BEKS data while a minimally coupled scalar field cannot.

Salgado *et al.* analysed an oscillating G model, induced by the oscillations of a non-minimally coupled scalar field ϕ , which also has a scalar potential $V(\phi)$ [8]. With a reasonable choice of initial conditions, they carefully fixed the parameters in the model and numerically investigated the possible oscillations in G , Hubble parameter H , energy density fractions and other quantities for spatially flat and open Friedmann–Robertson–Walker (FRW) models. The parameters were also fine tuned to the requirements of primeval nucleosynthesis. They showed in addition [9] that this scalar field can also account for the cosmological dark matter and up to 98% of the energy density of the Universe can be stored in the scalar field. In all these investigations the cosmological parameters like H or G oscillate at the present epoch. For example, in [8] these parameters are monotonic functions of time during the early stages of cosmic evolution and enters the oscillatory phase only in the later stages.

It is worthy of mention that the present rate of variation of G has a stringent upper bound imposed by the Viking radar echo experiment [10]. It was shown by Crittenden and Steinhardt [11] that there is an apparent conflict between these bounds and the value that is needed for periodicity in the galaxy distribution. In fact they argued that the nucleosynthesis constraints, consistent

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with these bounds, are very stringent on this oscillating G model unless a “fine tuning” of oscillation of the scalar field is assumed. In a recent work, González *et al.* [12] showed that with this restriction on the upper bound on the variation of G , one cannot have sufficient oscillation in order to explain the galactic periodicity as observed by the deep pencil beam survey. Furthermore, the luminosity–redshift relation of distant supernovae led to the conclusion that the present universe is accelerating its expansion [6] although, in order to facilitate the primeval nucleosynthesis, it must have been decelerating in the radiation era. These results might lead to the possibility that if the observed periodicity in the distribution of galaxies has to be attributed to the imprints of some temporal oscillations in some cosmological parameters, these oscillations should have taken place in the early stages of the evolution rather than recently.

Keeping this possibility in mind, we present an exact analytic model of a non–minimally coupled scalar field cosmology in which the oscillatory behaviour takes place in the past, namely during the radiation era. We start from Nordtvedt’s generalization of Brans Dicke (BD) theory [13] where the dimensionless BD parameter ω is taken to be a function of the scalar field ϕ [14]. We do not employ any additional scalar potential $V(\phi)$. The effective self–interaction is taken care of by the functional dependence of ω . With the matter distribution taken in the form of radiation, i.e., $\rho = 3p$, where p and ρ are the pressure and energy density of the cosmic fluid, it is found that there exists an exact analytical set of solutions for the field equations where ϕ , G , and H have an oscillatory phase. One important feature is that these oscillations die out in the radiation dominated period itself with G approaching a constant value and H becoming a monotonically decreasing function of time. The functional dependence of ω on ϕ may be carefully chosen so as to meet the observational requirements like the gravitational constant G stabilizing at the present value resulting in the transition to general relativity. This avoids the conflict between the bounds on variation of G and oscillations in G required for galactic periodicity.

The gravitational field equations in the generalized scalar tensor theory are too involved. So we adopt the following strategy. We effect a conformal transformation for the metric tensor components which results in a major simplification of the field equations [15] as they become more tractable. This transformed version is not “physical” in the sense that the geodesic equations are not valid in this version (Einstein’s frame). For a modern review, see Faraoni *et al.* [16]). But as we have the complete analytic solutions for the equations, we can transform the metric back to its original version (BD frame). We discuss all the cosmologically relevant functions in this physical version, and so the results obtained can be stated with confidence.

In section 2 we present the model and obtain the solutions. In section 3 the oscillatory behaviour is studied in detail. Finally, section 4 summarizes our conclusions and suggests directions of further investigation.

II. FIELD EQUATIONS AND FORMULATION OF THE MODEL

We start from the action

$$\bar{\mathcal{S}} = \frac{1}{16\pi G_0} \int \sqrt{-\bar{g}} \left[\phi \bar{\mathcal{R}} - \frac{\omega(\phi)}{\phi} \phi_{,\alpha} \phi^{,\alpha} + \bar{\mathcal{L}}_m \right] d^4x, \quad (1)$$

where $\bar{\mathcal{L}}_m$ is the matter lagrangian and G_0 the Newtonian gravitational constant. The dimensionless Brans–Dicke parameter ω is now assumed to be a function of the scalar field ϕ . This generalization of BD theory, proposed by Nordtvedt [14], includes a host of non–minimally coupled scalar tensor theories, suggested from different physical motivations and they are in fact the special cases of this ansatz.

To simplify the calculations we effect the conformal transformation

$$g_{\mu\nu} = \phi \bar{g}_{\mu\nu}, \quad (2)$$

so that the action looks like

$$\mathcal{S} = \frac{1}{16\pi G_0} \int \sqrt{-g} \left[\mathcal{R} - \frac{2\omega + 3}{\phi^2} \phi_{,\alpha} \phi^{,\alpha} + \mathcal{L}_m \right] d^4x. \quad (3)$$

Variables with and without an overhead bar are in the original version and the conformally transformed version respectively. For an FRW spacetime the gravitational field equations in the latter version are

$$3 \frac{\dot{a}^2}{a^2} + 3 \frac{k}{a^2} = 8\pi G_0 \rho + \frac{2\omega + 3}{4} \dot{\psi}^2, \quad (4)$$

$$2 \frac{\ddot{a}}{a} + \frac{k + \dot{a}^2}{a^2} = -8\pi G_0 p - \frac{2\omega + 3}{4} \dot{\psi}^2, \quad (5)$$

where a is the scale factor, k the curvature index having values 0 or ± 1 and $\psi = \ln(\frac{\phi}{\phi_0})$, ϕ_0 being a constant. The density and pressure of the cosmological perfect fluid are ρ and p , respectively. The wave equation for the scalar field is given by

$$(2\omega + 3)(\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi}) = 8\pi G_0 T - \dot{\omega}\dot{\psi}. \quad (6)$$

Here $T = \rho - 3p$ is the trace of the energy momentum tensor of the matter field.

The position of the first acoustic peak in the anisotropy power spectrum of the cosmic microwave background radiation strongly suggests that the Universe is spatially flat (see e.g., [17]), thereby we shall assume $k = 0$. Also we require to investigate the behaviour of the model at an early stage of evolution rather than at the present epoch, so we take the perfect fluid distribution as radiation, i.e., an equation of state $\rho = 3p$. This leads to a straightforward first integral of the wave equation as

$$(2\omega + 3)^{\frac{1}{2}} a^3 \dot{\psi} = A_1 \quad (7)$$

A_1 being a constant of integration. A combination of equation (4), (5) and (7) yields

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{A^2}{a^6} = 0, \quad (8)$$

where $A^2 = A_1^2/12$. After integrating once, equation (8) can be written as

$$u^2 = \frac{A^2}{a^4} + \frac{B_1}{a^2}, \quad (9)$$

where $u(a) = \dot{a}$ and B_1 is a constant of integration. This equation can again be integrated and it yields three results, depending on the signature of B_1 . In what follows we shall take $B_1 = B^2 > 0$. In this case the solutions for the scale factor can be written in terms of the transcendental function

$$a = \frac{1}{\sqrt{m^2 + a^2}} \left[2B(t + \tau) + m^2 \ln(a + \sqrt{m^2 + a^2}) \right], \quad (10)$$

where $m = A/B$ and τ is a constant of integration. From equation (7) and (9) one can easily obtain

$$\left(\frac{2\omega + 3}{12} \right)^{1/2} \frac{d\psi}{da} = \frac{m}{a\sqrt{m^2 + a^2}}, \quad (11)$$

From this equation ψ (or $\phi = e^{\psi}$) can be expressed as a function of a , if a definite functional form of $\omega = \omega(\phi)$ is chosen. In what follows, we take

$$2\omega + 3 = \frac{3}{4C} \frac{\phi^2}{(1 - \phi)[C(\phi - 1) + 1]}, \quad (12)$$

where C is a constant.

It deserves mention that Quevedo *et al.* [18] have shown that in order to account for the nonbaryonic dark matter as a non-minimally coupled scalar field, the effective ω parameter should be a ratio of quadratics in ϕ . The action integral (1) suggests that the gravitational coupling G is given as

$$G = \frac{G_0}{\phi}. \quad (13)$$

It is well-known that $\omega \rightarrow \infty$ is a necessary (although not sufficient [19]) requirement for the scalar tensor theories becoming indistinguishable from general relativity, where $G = G_0$. Our choice of ω indicates that for $\phi \rightarrow 1$, ω goes to the desired infinity limit and $G \rightarrow G_0$ (see Eq. (12)). Using the expression for ω in equation (12) and the relation $\psi = \ln(\phi/\phi_0)$, equation (11) can be integrated to yield

$$\phi = 1 - \frac{1}{C} \sin^2 \left[2C \ln \frac{a}{m + \sqrt{m^2 + a^2}} \right], \quad (14)$$

where a constant of integration is put equal to nought. It is clear from equation (14) that ϕ is not a monotonic function of the scale factor, but rather has an oscillation. But in order to understand the behaviour of the model, one has to find the expression of the scale factor in the original version of the theory, i.e., \bar{a} as given in equation (1), in place of a , because only in that version the theory retains the principle of equivalence and quantities carry their usual physical significance. Although we have an analytic form for a , but it is expressed in a transcendental way (equation (10)) and so we shall endeavour to investigate the actual behaviour of different relevant quantities by graphical representation.

III. OSCILLATORY BEHAVIOUR OF ϕ , G AND H

It is easy to recast the expression for the scalar field ϕ in equation (14) in terms of the original scale factor \bar{a} with the help of equation (2) as

$$\phi = 1 - \frac{1}{C} \sin^2 \left[2 C \ln \left[\frac{\bar{a} \sqrt{\phi}}{1 + \sqrt{1 + \phi \bar{a}^2}} \right] \right], \quad (15)$$

where we have chosen B to be $\frac{1}{2}$ and m to be 1 for the sake of computational simplicity. The expression for the effective gravitational constant $G(= \frac{1}{\phi})$, if the present value (G_0) is taken to be unity, is given as

$$G = \frac{1}{\phi} = \frac{1}{1 - \frac{1}{C} \sin^2 \left[2 C \ln \left[\frac{\bar{a} \sqrt{\phi}}{1 + \sqrt{1 + \phi \bar{a}^2}} \right] \right]}. \quad (16)$$

The Hubble parameter \bar{H} in this frame is given by,

$$\begin{aligned} \bar{H} &= \frac{\dot{\bar{a}}}{\bar{a}} = \frac{d}{dt} [\ln(a \sqrt{\phi})] = \frac{\dot{a}}{a} - \frac{1}{2} \frac{\dot{\phi}}{\phi} \\ &= \frac{1}{2 \phi^{\frac{3}{2}} \bar{a}^3} \left[\sqrt{1 + \phi \bar{a}^2} + \frac{\sin 2X}{1 - \frac{1}{C} \sin^2 X} \right], \end{aligned} \quad (17)$$

where $X = 2 C \ln \frac{\bar{a} \sqrt{\phi}}{1 + \sqrt{1 + \phi \bar{a}^2}}$. Now the scalar field ϕ , the Newtonian constant G and the Hubble parameter \bar{H} are all, in principle, known in terms of scale factor \bar{a} . But as these relations are involved and it is hardly possible to investigate the nature right from these expressions, we plot these quantities ϕ , G and \bar{H} against \bar{a} .

Before we actually plot these variables, the one arbitrary constant in equation (11), namely C , should be evaluated. As we are interested in the behaviour of the model in the early stages of the evolution, i.e., when the scale factor has small values, it would be useful to have a series expansion of equation (10). Along with the choice $B = \frac{1}{2}$ and $m = 1$, equation (10) can be approximated to

$$a = \left[\frac{3}{2}(t + \tau) \right]^{1/3}, \quad (18)$$

where the series is retained up to third order in a . Now with the help of this expression, the time evolution of the scalar field can be expressed as follows

$$\phi = 1 - \frac{1}{C} \sin^2 \left[\frac{2C}{3} \ln \left(1 + \frac{t}{\tau} \right) \right]. \quad (19)$$

Similarly from equations (17), (18) and (19) the time variation for the Hubble parameter \bar{H} is

$$\bar{H} = \frac{1}{3\tau(1 + \frac{t}{\tau})} \left[1 + \frac{1}{1 - (\frac{2t}{3\tau})^2} \sin^2 \left(\frac{2C}{3\tau} t \right) \right]. \quad (20)$$

Clearly the part within the square bracket produces an oscillatory behaviour in \bar{H} . The first part $\frac{1}{3\tau(1 + \frac{t}{\tau})}$ primarily behaves as t^{-1} , which we expect to get towards the end of the radiation regime. We designate the functional dependence of the Hubble parameter at the end of radiation era as

$$\bar{H}_r = \frac{1}{3\tau(1 + \frac{t}{\tau})}. \quad (21)$$

Keeping in mind the relation $1 + z = a^{-1}$, it is straight forward to relate z_r , the usual redshift of the expanding universe in the radiation era, to the redshift z , corrected for oscillation [5,7], by

$$\frac{d\bar{z}}{d\bar{z}_r} \sim \frac{d\bar{z}}{\bar{z}_r} = \frac{\bar{H}}{\bar{H}_r} = 1 + \frac{1}{[1 - (\frac{2t}{3\tau})^2]} \sin^2 \left[\frac{2C}{3\tau} t \right]. \quad (22)$$

From equation (22) it is evident that the frequency of oscillation is given by

$$\nu = \frac{2}{3} \frac{C}{\tau}. \quad (23)$$

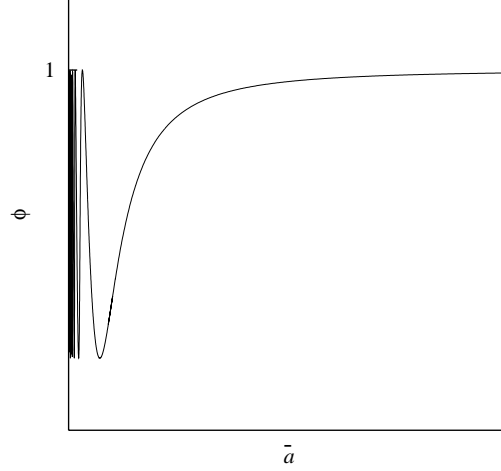


FIG. 1. Plot of the Brans Dicke field ϕ against the original scale factor \bar{a}

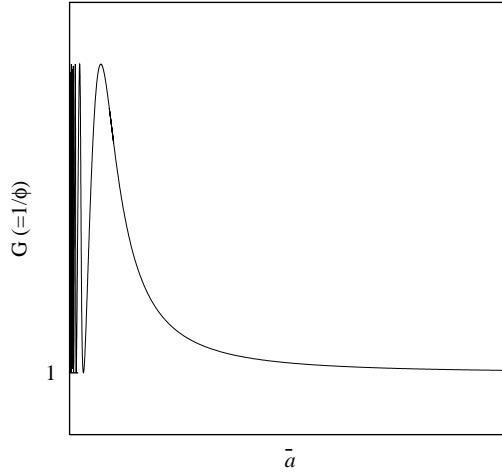


FIG. 2. Plot of the Gravitational Constant G against the original scale factor \bar{a}

where as the amplitude of oscillation depends on time as

$$\mathcal{A} = \frac{1}{1 - \frac{4t^2}{9\tau^2}}. \quad (24)$$

We now plot ϕ , G and \bar{H} , all against \bar{a} from equations (15), (16) and (17) respectively with a value $C = 5$.

Figure 1 shows that ϕ has an oscillation for very small \bar{a} and then attains a value of unity. From figure 2 one can find that G also has an initial oscillation and then quickly settles down to a value close to its present value (G_0 is taken to be unity in the units in which the equations are written). \bar{H} too has an oscillation (see figure 3), about a steadily decreasing mean value and then becomes a monotonically decreasing function of \bar{a} . This is a desirable feature as in the radiation paradigm we need a decelerating universe ($q > 0$). Figure 4 shows that Ω_ϕ , the dimensionless density parameter corresponding to the scalar field, also has an initial oscillation, and then approaches the value zero in the early radiation era. This is safely below the permitted

upper bound of $\Omega_\phi < 0.2$ conducive for a successful nucleosynthesis [20].

One point to note here is that the nature (some oscillation at an early stage and then monotonic behaviour) of these graphs are not really too sensitive to the value of C , only the frequency and amplitude of oscillations get modified with the choice of C .

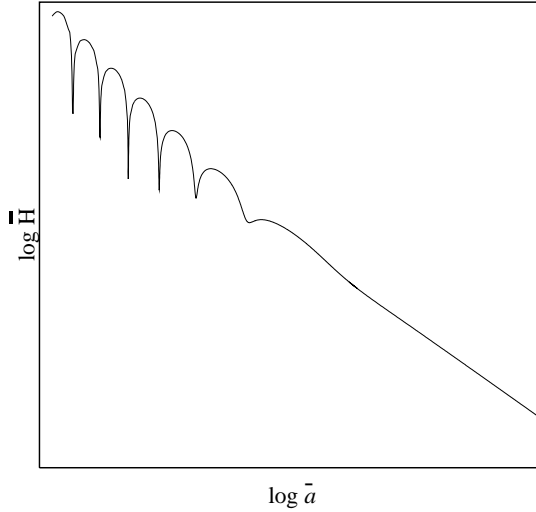


FIG. 3. Plot of the Hubble parameter \bar{H} (original unit) against the original scale factor \bar{a} in logarithmic scale

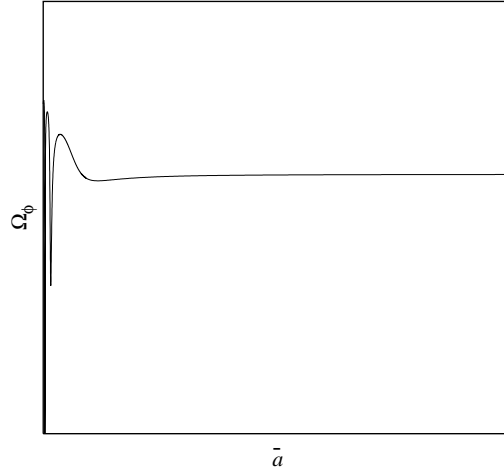


FIG. 4. Plot of density parameter Ω_ϕ against the original scale factor \bar{a}

IV. CONCLUDING REMARKS

In order to have a compromise between the periodicity in the galaxy distribution and the cosmological principle, one might require a temporal periodicity in the cosmological model resulting in an apparent spatial periodicity. An oscillating G model is indeed one of the most viable options in this connection.

We have resorted to Nordtvedt's scalar tensor theory to build a model in which ϕ , G and H present oscillations but confined to the radiation-dominated era of cosmic expansion. Owing to the fact that the dark-matter component does not interact with the radiation field it clusters with the periodicity of the oscillations. Later on, during the matter era the baryonic component is free to fall in the potential wells created by the dark component. Our solutions are analytical, but a bit involved -Eqs. (15)–(17). This is why we have depicted their behaviour with expansion. Aside from being compatible with primordial nucleosynthesis

bounds our model has the advantage that the BD parameter diverges for large \bar{a} , therefore no conflict with local measurements of ω arises [10]. So one really has wider options to account for the periodicity of the galaxy distributions as an imprint of a temporal oscillation in G and H .

Obviously, since the oscillations do not extend to the matter era the model presented in this paper cannot account for the alleged periodicity in length of the solar year, supposedly derived from periodic structures found in coral fossils and marine bivalves [21]. However, it should be noted that such periodicity may well be purely biological in origin and therefore unconnected to alleged oscillations of the solar year.

It also deserves mention that the functional dependence of ω on ϕ taken up in this work -Eq. (12)- is by no means unique. There could well be possibilities of $\omega = \omega(\phi)$ other than the one adopted here. This indeed gives a flexibility for improvisation if required to fit in other observations such as the late time acceleration of the Universe.

In fact it will be worthwhile to figure out the correct $\omega = \omega(\phi)$ which serves all these purpose, namely drives an oscillation at some stage, gives perfect ambience for nucleosynthesis, and generates sufficient negative pressure in the later stage so that an accelerated expansion for the present universe could be explained.

From the figures it can be seen that our model does not produce enough number of oscillations, i.e., more rings of galaxies are observed than oscillations our model is able to provide. This is why our model cannot be viewed as fully accounting for the inhomogeneous distribution of galaxies. We believe, however, it may serve as a starting point for more detailed models that overcome this limitation. A lesser difficulty refers to the fact that our universe seems to be accelerating its expansion today (see e.g. [22] and references therein) while our predicts deceleration at the present time. However, this may be solved by introducing some quintessence scalar field such that its contribution to the total energy density becomes relevant only recently [23]. We have chosen not to go into that at this stage in order to focus on the problem of galaxy distribution.

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